The supply chain of enterprise software

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Appendix A

Proof of Proposition 4.3:

First, consider the case when $\bar{c} < \upsilon\theta_L$. The profits of the software vendor equal $\bar{c}$ under fixed-fee licensing and $\frac{\theta_H+\theta_L}{2\theta_H}\bar{c}$ under pay-per-use licensing. Because $\frac{\theta_H+\theta_L}{2\theta_H}\bar{c} < \frac{\theta_H+\theta_H}{2\theta_H}\bar{c} = \bar{c}$, the software vendor prefers fixed-fee over pay-per-use licensing.

Next, consider the case when $\upsilon\theta_L \leq \bar{c} \leq \upsilon\theta_H$. The profits of the software vendor equal $\upsilon\theta_L$ under fixed-fee licensing and $\frac{\theta_H+\theta_L}{2\theta_H}\bar{c}$ under pay-per-use licensing. Rewriting $\upsilon\theta_L \geq \frac{\theta_H+\theta_L}{2\theta_H}\bar{c}$ gives $\bar{c} \leq \frac{2\upsilon\theta_L}{(\theta_L/\theta_H)+1}$. Because (i) $\frac{2\upsilon\theta_L}{(\theta_L/\theta_H)+1} > \upsilon\theta_L$ and (ii) $\frac{2\upsilon\theta_L}{(\theta_L/\theta_H)+1} < \frac{\upsilon\theta_L+\upsilon\theta_H}{(\theta_L/\theta_H)+1} = \frac{\upsilon\theta_H\{(\theta_L/\theta_H)+1\}}{(\theta_L/\theta_H)+1} = \upsilon\theta_H$, we conclude that $\frac{2\upsilon\theta_L}{(\theta_L/\theta_H)+1}$ is within the interval $[\upsilon\theta_L, \upsilon\theta_H]$, as required.

Finally, consider the case when $\bar{c} > \upsilon\theta_H$. The profits of the software vendor equal $\upsilon\theta_L$ under fixed-fee licensing and $\frac{1}{2}(\theta_H + \theta_L)\upsilon$ under pay-per-use licensing. Because $\frac{1}{2}(\theta_H + \theta_L)\upsilon > \frac{1}{2}(\theta_L + \theta_L)\upsilon = \upsilon\theta_L$, the software vendor finds pay-per-use licensing more profitable than fixed-fee licensing, which completes the proof. □

Proof of Proposition 4.4:

First, consider the case when $\bar{c} < \upsilon\theta_L$. The profits of the software vendor equal $\bar{c}$ under fixed-fee licensing and $\frac{\theta_H+\theta_L}{2\theta_H}\bar{c}$ under pay-per-use licensing. Because $\frac{\theta_H+\theta_L}{2\theta_H}\bar{c} < \frac{\theta_H+\theta_H}{2\theta_H}\bar{c} = \bar{c}$, the software vendor prefers fixed-fee over pay-per-use licensing.

Second, consider the case when $\upsilon\theta_L \leq \bar{c} < \frac{1}{2}\upsilon\theta_H$. The profits of the software vendor equal $\frac{\theta_H-\bar{c}/\upsilon}{\theta_H-\theta_L}\bar{c}$ under fixed-fee licensing and $\frac{\theta_H+\theta_L}{2\theta_H}\bar{c}$ under pay-per-use licensing. Because $\frac{\theta_H-\bar{c}/\upsilon}{\theta_H-\theta_L}\bar{c} > \frac{\theta_H-(\frac{1}{2}\upsilon\theta_H)/\upsilon}{\theta_H-\theta_L}\bar{c} \geq \frac{\theta_H-(\theta_L)^2/\theta_H}{2(\theta_H-\theta_L)}\bar{c} = \frac{\theta_H+\theta_L}{2\theta_H}\bar{c}$,

the software vendor finds fixed-fee licensing more profitable than pay-per-use licensing.

Third, consider the case when $\frac{1}{2}\upsilon\theta_H \leq \bar{c} \leq \upsilon\theta_H$. The profits of the software vendor equal $\frac{(\theta_H)^2}{4(\theta_H-\theta_L)}\upsilon$ under fixed-fee licensing and $\frac{\theta_H+\theta_L}{2\theta_H}\bar{c}$ under pay-per-use licensing. Rewriting $\frac{(\theta_H)^2}{4(\theta_H-\theta_L)}\upsilon \geq \frac{\theta_H+\theta_L}{2\theta_H}\bar{c}$ gives $\bar{c} \leq \frac{(\theta_H)^3}{2((\theta_H)^2-(\theta_L)^2)}\upsilon$. Because (i) $\frac{(\theta_H)^3}{2((\theta_H)^2-(\theta_L)^2)}\upsilon < \frac{(\theta_H)^3}{2((\theta_H)^2-\frac{1}{4}(\theta_H)^2)}\upsilon < \upsilon\theta_H$ and (ii) $\frac{(\theta_H)^3}{2((\theta_H)^2-(\theta_L)^2)}\upsilon > \frac{(\theta_H)^3}{2(\theta_H)^2}\upsilon = \frac{1}{2}\upsilon\theta_H$, we conclude that $\frac{(\theta_H)^3}{2((\theta_H)^2-(\theta_L)^2)}\upsilon$ is within the interval $[\frac{1}{2}\upsilon\theta_H, \upsilon\theta_H]$, as required.

Finally, consider the case when $\bar{c} > \upsilon\theta_H$. The profits of the software vendor equal $\frac{(\theta_H)^2}{4(\theta_H-\theta_L)}\upsilon$ under fixed-fee licensing and $\frac{1}{2}(\theta_H+\theta_L)\upsilon$ under pay-per-use licensing. Because $\frac{1}{2}(\theta_H+\theta_L)\upsilon = \frac{(\theta_H)^2-(\theta_L)^2}{2(\theta_H-\theta_L)}\upsilon > \frac{(\theta_H)^2-\frac{1}{4}(\theta_H)^2}{2(\theta_H-\theta_L)}\upsilon > \frac{(\theta_H)^2}{4(\theta_H-\theta_L)}\upsilon$, the software vendor prefers pay-per-use over fixed-fee licensing, which completes the proof. □